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DISPLACEMENT WAVES FOR STRAIN LOCALIZATION IN THE STRETCHING OF A STRIP WITH ELASTOPLASTIC SEAMS

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The books [1-3] deal with experimental and theoretical investigations of displacement waves in structurally heterogeneous media under plastic deformation.

Numerical modeling of displacement waves has been considered in [4-6]. The vortex nature of the velocities under dynamical deformation of a medium consisting of rigid nondeformable elements with specified interaction forces between them has been studied in [4]. Strain nonuniformity and displacement waves in a medium consisting of elastoplastic elements and seams whose plastic properties differ from those of the main elements have been studied in [5, 6]. A survey of the investigations [4-6] and their developments is given in [3].

Here we present the results of numerical modeling of displacement waves and strain localization in the stretching of a strip consisting of rigid (nondeformable) blocks interspersed with elastoplastic layers.

1. Rigidity Equations for an Elastoplastic Seam. We consider two rigid (nondeformable) blocks connected by an elastoplastic seam (Fig. 1a). We assume that the velocity field of the blocks and the seam is plane. Let $\mathbf{u}_\pm$ be the velocities of motion of the block centers O_+ and O_-, $\omega_\pm$ the angular velocities of the blocks, and $\mathbf{F}_\pm$ and $\mathbf{M}_\pm$ the forces and torques about the centers O_+ and O_- exerted by the blocks on the seam. We agree to call the dependences of $\mathbf{F}_\pm$ and $\mathbf{M}_\pm$ on $\mathbf{u}_\pm$ and $\omega_\pm$ the rigidity equations of the seam.

For the seam deformation model we use the couple-free model in [7]. According to this model the seam is represented by a layer of rectangular elements (Fig. 1a, b) with rigidity equations

$$\begin{aligned} \mathbf{p}_+^\alpha - \mathbf{p}_-^\alpha &= D^{\alpha\beta}\tau(\mathbf{u}_+^\beta + \mathbf{u}_-^\beta) + 2\chi^\alpha, \\ \mathbf{p}_+^\alpha + \mathbf{p}_-^\alpha &= C^{\alpha\beta}\tau(\mathbf{u}_+^\beta - \mathbf{u}_-^\beta) + 2(\mathbf{p}_\times^\alpha)^0, \end{aligned} \tag{1.1}$$

where

$$\mathbf{u}_\pm^\alpha = \mathbf{u}^\alpha|_{\xi^\alpha=\pm1}, \qquad \mathbf{p}_\pm^\alpha = (\hat\sigma^{\alpha\beta}\sqrt{g}\,\hat{\mathbf{\Im}}_\beta)|_{\xi^\alpha=\pm1}; \tag{1.2}$$

$\hat\sigma^{\alpha\beta}$ are the components of the stress tensor in the oblique coordinates $\xi^1, \xi^2 \in [-1, 1]$ connected with the seam element (Fig. 1b), $\sqrt{g} = |\hat{\mathbf{\Im}}_1 \times \hat{\mathbf{\Im}}_2|$ ($\hat{\mathbf{\Im}}_\alpha$ are the basis vectors of the frame ξ^α), and τ is a time step ($\tau = 1$ everywhere below). In each iteration, $D^{\alpha\beta}$, $C^{\alpha\beta}$, χ^α, and $(\mathbf{p}_\times^\alpha)^0$ are known constant quantities within the confines of an element, which are corrected in transition from one iteration to another by the procedure expounded in detail in [7].

At the interfaces of the blocks and elements of the seam the normal components of the velocities are assumed to be continuous:

$$(\mathbf{u}_\pm^2 - \mathbf{u}_\pm - \omega_\pm \times \mathbf{r}_\pm)\cdot\hat{\mathbf{\Im}}^2\big|_{\xi^2=\pm1} = 0. \tag{1.3}$$

The tangential components of the velocities in the couple-free model [7] can be discontinuous. In the first iteration it is assumed that

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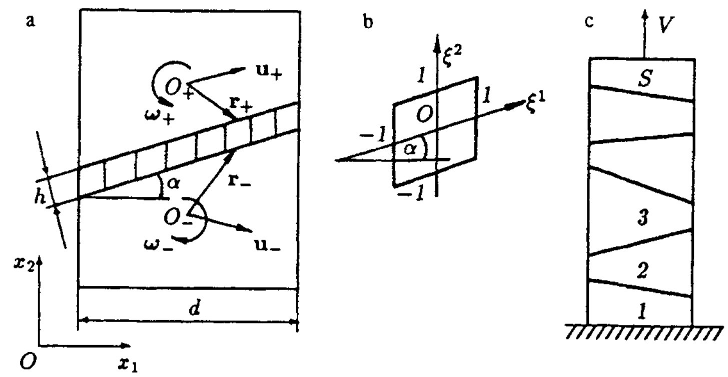

Fig. 1

$$(\mathbf{u}^2_\pm - \mathbf{u}_\pm - \omega_\pm \times \mathbf{r}_\pm) \cdot \hat{\mathfrak{I}}_1 \Big|_{\xi^2=\pm1} = 0. \tag{1.4}$$

In subsequent iterations, the condition (1.4) is preserved only at those faces where the magnitude of the shearing stresses does not exceed the yield point τ_*. At the other faces the condition (1.4) is replaced by the condition that the magnitude of the shearing stresses is equal to the yield point. Moreover, the sign of the shearing stresses remains the same as that in the condition (1.4). Thus, in the general case, there will be faces of the seam elements where the conditions (1.4) hold, together with faces where conditions of the form

$$(\mathbf{p}^2_+ \cdot \hat{\mathfrak{I}}_1)\Big|_{\xi^2=+1} = \pm\tau_* |\hat{\mathfrak{I}}_1|^2\Big|_{\xi^2=+1}, \quad (\mathbf{p}^2_- \cdot \hat{\mathfrak{I}}_1)\Big|_{\xi^2=-1} = \pm\tau_* |\hat{\mathfrak{I}}_1|^2\Big|_{\xi^2=-1}. \tag{1.5}$$

are satisfied. In the process of iterations the conditions of the form (1.5) are preserved at those faces where the power dissipated in slipping of these faces relative to the blocks is nonnegative. At the other faces the conditions of the form (1.5) are replaced by the conditions (1.4).

From Eqs. (1.1)-(1.5) and the continuity conditions for the stresses and velocities at the common faces of adjacent seam elements it follows that

$$\begin{aligned}
\mathbf{p}^1_i &= A^{(1)}_{i+1/2}\mathbf{u}^1_i + A^{(2)}_{i+1/2}\mathbf{u}^1_{i+1} + \Phi^{(1)}_{i+1/2}\mathbf{v}_- + \Phi^{(2)}_{i+1/2}\mathbf{v}_+ + \varphi^{(1)}_{i+1/2},\\
\mathbf{p}^1_{i+1} &= A^{(3)}_{i+1/2}\mathbf{u}^1_i + A^{(4)}_{i+1/2}\mathbf{u}^1_{i+1} + \Phi^{(3)}_{i+1/2}\mathbf{v}_- + \Phi^{(4)}_{i+1/2}\mathbf{v}_+ + \varphi^{(2)}_{i+1/2};
\end{aligned} \tag{1.6}$$

$$\begin{aligned}
(\mathbf{p}^2_-)_{i+1/2} &= B^{(1)}_{i+1/2}\mathbf{u}^1_i + B^{(2)}_{i+1/2}\mathbf{u}^1_{i+1} + \Psi^{(1)}_{i+1/2}\mathbf{v}_- + \Psi^{(2)}_{i+1/2}\mathbf{v}_+ + \psi^{(1)}_{i+1/2},\\
(\mathbf{p}^2_+)_{i+1/2} &= B^{(3)}_{i+1/2}\mathbf{u}^1_i + B^{(4)}_{i+1/2}\mathbf{u}^1_{i+1} + \Psi^{(3)}_{i+1/2}\mathbf{v}_- + \Psi^{(4)}_{i+1/2}\mathbf{v}_+ + \psi^{(2)}_{i+1/2}, \quad i = 0, 1, \dots, N-1.
\end{aligned} \tag{1.7}$$

Here, N is a number of the elements of the seam, $\mathbf{p}^1_i$ and $\mathbf{u}^1_i$ are the stress and velocity vectors at the faces $\xi^1 = \pm1$ of the elements, $\mathbf{v}_+$ and $\mathbf{v}_-$ are the vectors:

$$\mathbf{v}_+ = (u^1_+,\ u^2_+,\ \omega_+), \qquad \mathbf{v}_- = (u^1_-,\ u^2_-,\ \omega_-);$$

and u^k_+ and u^k_- (k = 1, 2) are the Cartesian components of the vectors $\mathbf{u}_+$ and $\mathbf{u}_-$.

Below, we consider the deformation of seams with zero stresses at the end faces of the seams. In this case

$$\mathbf{p}^1_0 = \mathbf{p}^1_N = 0. \tag{1.8}$$

The system of equations (1.6) and (1.8) defines the dependence of the vectors $\mathbf{u}^1_i$ on $\mathbf{v}_-$ and $\mathbf{v}_+$:

$$\mathbf{u}^1_i = D^-_i\mathbf{v}_- + D^+_i\mathbf{v}_+ + \mathbf{d}_i, \quad i = 0, 1, \dots, N. \tag{1.9}$$

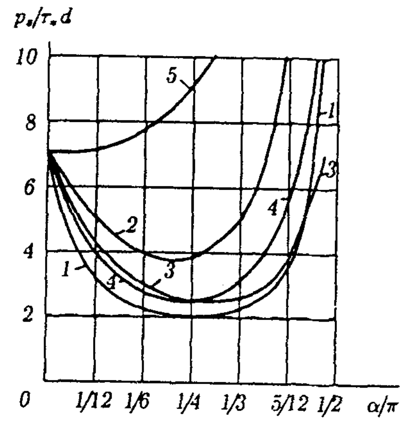

Fig. 2

The matrices $D^{\pm}_i$ and the vectors $\mathbf{d}_i$ can be calculated by a double sweep (modified Gaussian elimination) procedure in two stages. In the first stage Eqs. (1.6) and (1.8) are transformed into the system of equations

$$\mathbf{u}_i^1 = G_i \mathbf{u}_{i+1}^1 + H_i^- \mathbf{v}_- + H_i^+ \mathbf{v}_+ + \mathbf{g}_i, \quad i = 0, 1, \ldots, N-1.$$

Here, the matrices G_i, H^-_i, and H^+_i and the vectors g_i are calculated by the recursion formulas:

$$G_0 = -(A^{(1)}_{1/2})^{-1} A^{(2)}_{1/2}, \qquad \mathbf{g}_0 = -(A^{(1)}_{1/2})^{-1} \varphi^{(1)}_{1/2},$$

$$H_0^- = -(A^{(1)}_{1/2})^{-1} \Phi^{(1)}_{1/2}, \qquad H_0^+ = -(A^{(1)}_{1/2})^{-1} \Phi^{(2)}_{1/2},$$

$$G_{i+1} = L_i^{(1)} A^{(2)}_{i+3/2}, \qquad L_i^{(1)} = (A^{(3)}_{i+1/2} G_i + A^{(4)}_{i+1/2} - A^{(1)}_{i+1/2})^{-1},$$

$$H^-_{i+1} = L_i^{(1)} (\Phi^{(1)}_{i+3/2} - \Phi^{(3)}_{i+1/2} - A^{(3)}_{i+1/2} H_i^-),$$

$$H^+_{i+1} = L_i^{(1)} (\Phi^{(2)}_{i+3/2} - \Phi^{(4)}_{i+1/2} - A^{(3)}_{i+1/2} H_i^+),$$

$$\mathbf{g}_{i+1} = L_i^{(1)} (\varphi^{(1)}_{i+3/2} - \varphi^{(2)}_{i+1/2} - A^{(3)}_{i+1/2} \mathbf{g}_i), \qquad i = 0, 1, \ldots, N-1.$$

In the second stage the equations

$$\mathbf{u}^1_{N-1} - (G_{N-1} \mathbf{u}^1_N + H^+_{N-1} \mathbf{v}_- + H^+_{N-1} \mathbf{v}_+ + \mathbf{g}_{N-1}) = 0,$$

$$A^{(3)}_{N-1/2} \mathbf{u}^1_{N-1} + A^{(4)}_{N-1/2} \mathbf{u}^1_N + \Phi^{(3)}_{N-1/2} \mathbf{v}_- + \Phi^{(4)}_{N-1/2} \mathbf{v}_+ + \varphi^{(2)}_{N-1/2} = 0.$$

Define D^-_N, D^+_N, d_N:

$$D^-_N = L_N^{(2)} [H^-_{N-1} + (A^{(3)}_{N-1/2})^{-1} \Phi^{(3)}_{N-1/2}], \quad L_N^{(2)} = -[(A^{(3)}_{N-1/2})^{-1} A^{(4)}_{N-1/2} + G_{N-1}],$$

$$D^+_N = L_N^{(2)} [H^+_{N-1} + (A^{(3)}_{N-1/2})^{-1} \Phi^{(4)}_{N-1/2}], \quad \mathbf{d}_N = L_N^{(2)} [\mathbf{g}_{N-1} + (A^{(3)}_{N-1/2})^{-1} \varphi^{(2)}_{N-1/2}],$$

are used to calculate D^-_i, D^+_i, and $\mathbf{d}_i$ (i = N, N − 1, N − 2,..., 1, 0)

$$D_i^- = G_i D^-_{i+1} + H_i^-, \qquad D_i^+ = G_i D^+_{i+1} + H_i^+,$$

$$\mathbf{d}_i = G_i \mathbf{d}_{i+1} + \mathbf{g}_i, \qquad i = N-1, N-2, \ldots, 1, 0.$$

We introduce the notation

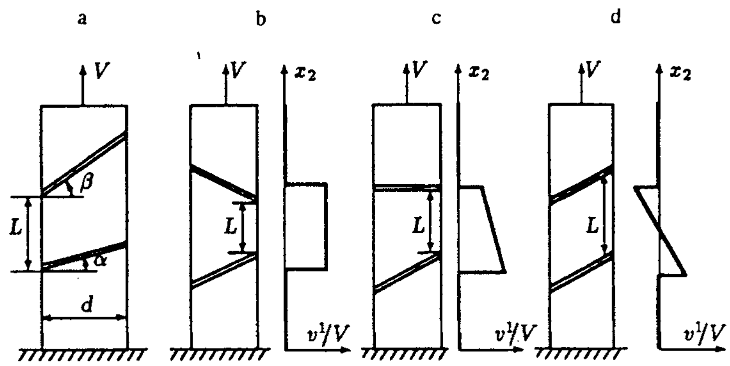

Fig. 3

$$F_{\pm}^{k} = \sum_{i=0}^{N-1} (\mathbf{p}_{\pm}^{2})_{i+1/2} \cdot \mathbf{e}^{k}, \qquad M_{\pm} = \sum_{i=0}^{N-1} (\mathbf{r}_{\pm} \times \mathbf{p}_{\pm}^{2})_{i+1/2} \cdot \mathbf{e}^{3}, \tag{1.10}$$
$$\mathbf{Q}_{\pm} = (F_{\pm}^{1}, F_{\pm}^{2}, M_{\pm}),$$

where $\mathbf{e}^k$ (k = 1, 2, 3) are the basis vectors of the Cartesian coordinate system.

From (1.7), (1.9), and (1.10) it follows that the rigidity equations for the seam can be written in the form

$$\mathbf{Q}_{-} = C^{(1)}\mathbf{v}_{-} + C^{(2)}\mathbf{v}_{+} + \mathbf{c}^{(1)}, \qquad \mathbf{Q}_{+} = C^{(3)}\mathbf{v}_{-} + C^{(4)}\mathbf{v}_{+} + \mathbf{c}^{(2)}. \tag{1.11}$$

Here the matrices $C^{(k)}$ (k = 1, ..., 4) and the vectors $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ are related in an obvious way to the matrices and vectors appearing in Eqs. (1.7) and (1.9).

2. Equilibrium Conditions for Rigid Blocks. We denote the number of rigid (nondeformable) block forming the strip by S (Fig. 1c), and the vectors (u_k^1, u_k^2, ω_k) by $\mathbf{v}_k$ (k = 1, 2,..., S), where u_k^1 and u_k^2 are the Cartesian components of the velocity of the k-th block center, and ω_k is the angular velocity of this block.

We assume that stretching of the strip is specified in the form of the conditions that the block k = 1 is immovable, while the block k = S moves translationally with a specified velocity V, and hence,

$$\mathbf{v}_1 = (0, 0, 0), \qquad \mathbf{v}_S = (0, V, 0). \tag{2.1}$$

If the coefficients of the rigidity equations (1.11) are calculated by the method presented above for each seam, the equilibrium conditions for the blocks (the sum of the forces acting on a block and the sum of the moments of these forces are equal to zero) can be formulated in the form of the following equations

$$A_k\mathbf{v}_{k-1} + B_k\mathbf{v}_k + C_k\mathbf{v}_{k+1} = \mathbf{f}_k, \quad k = 2, 3, \ldots, S-1, \tag{2.2}$$

the coefficients of which are related in an obvious way to those of the rigidity equations for the seams. The solution of the system of equations (2.1) and (2.2) can be computed by the double sweep method [8].

3. Solution Scheme for the Problem of Stretching of a Strip with Seams. The solution is constructed in time steps. The stresses and strain rates in a time step are calculated iteratively. The calculation of an iteration consists of two main stages:

1) Form the coefficients of the Eqs. (2.2). In the first iteration of the first time step this is carried out on the basis of the assumption that all elements of the seams deform elastically and there is no slipping of the seams relative to the blocks. In subsequent iterations this is carried out on the basis of Eqs. (1.1) and the slipping conditions of the seams defined by the strain rates and the stresses obtained in the previous iteration.

2) Solve Eqs. (2.1) and (2.2) by the double sweep method, calculate the strain rates of the seam from the previously determined velocities of the rigid blocks and the stresses at the interfaces between the seams and the blocks, and correct the coefficients of the seams on the basis of the results obtained.

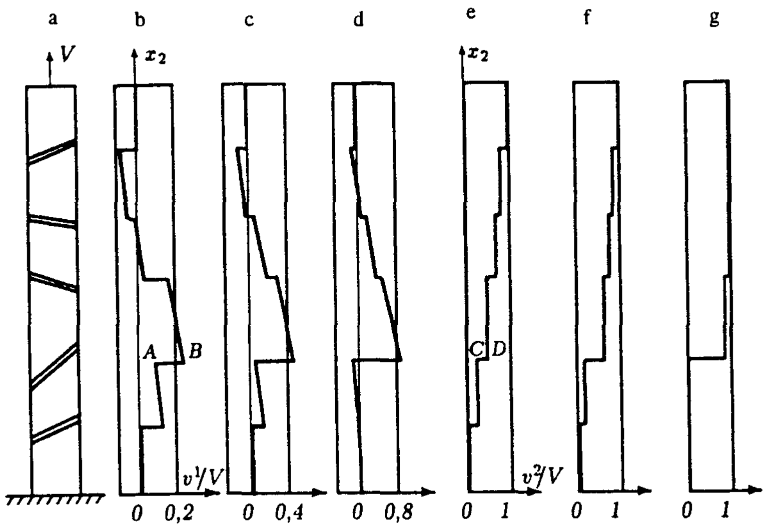

Fig. 4

4. Limit State of the Strip. Below, the deformation of the seams is assumed to be ideal elastoplastic. In this case the elongation cannot exceed the value for which the limit state occurs in the strip (deformation at constant stresses). We will call the limiting value of the tensile force the limiting load. We denote it by P_{sd}.

The stressed state

$$\sigma_{22} = 2\tau_*, \qquad \sigma_{11} = \sigma_{12} = 0$$

is statically admissible for stretching of the strip with seams. Therefore [9], we have the lower bound on the limiting load

$$P_s \geq 2\tau_* d, \tag{4.1}$$

where d is the width of the strip.

The velocity field in a strip with a given system of seams is kinematically possible for a strip which contains more seams in addition to the given system of seams. Therefore [9], the limiting load of the strip with a given system of seams is the upper bound for the limiting load of the strip which contains additional seams. with the given system of seams.

It follows from numerical calculations that the limiting loads and localization of strains in the limiting state for stretching of a strip with seams depend on the restrictions imposed on the transverse displacements and rotations of the rigid blocks. Below, two limiting cases will be considered: stretching under restrictions excluding the possibility of transverse displacements and rotations of the blocks, and stretching without any restrictions imposed on the transverse displacements and rotations. Stretching under the conditions provided for the contact of the strip surfaces with absolutely smooth and absolutely rigid slabs is an example of stretching of the strip without transverse displacements and rotations of blocks.

5. Limiting Loads and Strain Localization in Stretching without Transverse Displacements and Rotation of the Blocks. The simplest case of stretching of a strip without transverse displacements and rotations of the blocks is the stretching of the strip consisting of two blocks separated by a seam. In this case, according to the conditions (2.1) one of the blocks is immovable, while the other moves translationally along the strip axis.

Curve 5 in Fig. 2 represents the dependence of the limiting load of a two-block strip on angle of inclination of the seam (angle α in Fig. 1a). From now on the thickness h of the seam is assumed in calculations to be equal to one-tenth the width d of the strip (Fig. 1a).

For stretching without transverse displacements and rotations the deformation conditions for each seam of the strip are the same as those for stretching without transverse displacements and rotations will correspond to curve 5 in Fig. 2 if the angle α in this figure is interpreted as the least of the moduli of the angles of inclination of the seams in the strip.

In the limiting state of the strip in stretching without transverse displacements and rotations the strain is localized in the seam having the least angle of inclination taken absolutely. If there are several seams having the minimum absolute value of the angle of inclination, either all these streaks or only some of them can deform in the limiting state.

6. Limiting Loads for Stretching of a Strip with Two Seams. The limiting loads for stretching of a strip with two seams without any restrictions imposed on transverse displacements and rotations of the blocks between the seams depend on the orientation of the seams (on the angles α and β) and the distance L between the seams (Fig. 3a).

The dependence of the limiting load on orientation of the seams is shown in Fig. 2 as the dependence of the limiting load on the angle α for $\beta = -\alpha$, α, and 0 (curves 1-3). The region intercepted by curves 1 and 3 corresponds to the limiting loads for $-\alpha \le \beta \le 0$, while that intercepted by curves 3 and 2 corresponds to the limiting loads for $0 \le \beta \le \alpha$. The calculations were carried out for $h/d = 0, 1$ and $L/d = 1$.

The limiting loads for $\beta = -\alpha$ are independent of the distance L between the streaks. The dependence of the limiting loads on L for $\beta \neq -\alpha$ is illustrated in Fig. 2 by curve 4 corresponding to the limiting loads for $\beta = \alpha$ and $L/d = 2$. The region between curves 1 and 4 corresponds to the limiting load for $-\alpha \le \beta \le \alpha$ and $L/d = 2$. It is considerably smaller than the region between curves 1 and 2 which corresponds to the limiting loads for $-\alpha \le \beta \le \alpha$ and $L/d = 1$. It follows from numerical calculations that the region of the limiting loads diminishes, shrinking to curve 1, as the ratio L/d increases. Therefore, for a sufficiently large distance between the seams the limiting loads can be determined from curve 1 if the angle α is identified with the largest of the moduli of the angles of inclination of the seams.

The dashed line in Fig. 2 represents the limiting load for the stretching of a homogeneous ideal plastic strip having the yield point τ_*. According to (4.1), it is the lower bound of the limiting loads for stretching of the strip with seams. From a comparison between the dashed line and curve 1 it follows that in the case of stretching of the plate with two seams sufficiently far apart with angles of inclination satisfying the inequality

$$\frac{\pi}{6} \le \max(|\alpha|, |\beta|) \le \frac{\pi}{3}, \tag{6.1}$$

the limiting loads differ from the lower bound by no more than 15%.

The limiting loads for a strip with two seams are the upper bounds for limiting loads for a strip having more than two seams. Therefore, the difference between the limiting loads for strip with any number of streaks (greater than two) and the lower bound for the limiting loads will not exceed 15% in the case when the maximum absolute value of the angle of inclination of the seams satisfies the inequality (6.1).

From a comparison between curves 1 and 5 in Fig. 2 it follows that the possibility of transverse displacements decreases the limiting loads considerably. Therefore, upon stretching the strip with seams transverse displacements will always occur if there are no restrictions to prevent their occurrence. The transverse velocities v^1 of the strip axis in the limiting state of the strip with two seams for $\beta = -\alpha$, 0, and α are shown in Figs. 3b-d respectively.

7. Transverse Displacement Waves and Strain Localization in the Stretching of a Strip with Three and Five Seams. The authors have performed a great many numerical experiments on the limiting loads, transverse displacement waves, and strain localization for the stretching of strips with three and five seams. Seams with angles of inclination relative to the x_1 axis less than or equal to $\pi/4$ in absolute value were considered. The angles of inclination of the seams were chosen randomly using a random number generator. The calculations were carried out for $L/d = 1$.

It can be concluded from the results of numerical experiments in Sec. 6 that the limiting loads for stretching of a strip with two seams are not smaller than those corresponding to curve 1 in Fig. 2 if the angle α in this figure is identified with the largest absolute value of the angles of inclination of the seams. From the results of numerical experiments on the stretching of strips with three and five seams it follows that this property of the limiting loads is also preserved for the stretching of strips with three and five streaks. In all the experiments performed, the limiting loads were not less than those described by curve 1 in Fig. 2, provided the angle α in this figure is assumed to be the largest absolute value of the angles of inclination of the seams.

The limiting loads obtained in all the experiments differed slightly from those corresponding to curve 1 in Fig. 2. Therefore, the loads corresponding to curve 1 in Fig. 2 may be considered as the limiting loads for stretching of a strip having more than two seams provided the angle α in this figure is identified with the largest absolute value of the angles of inclination of the seams. In the stretching of a strip with three and five seams the strain of the strip in the limiting state is localized in one or two seams. Localization occurs either in the seam with the largest angle of inclination or in this seam and in one other.

In the stretching of strips with three and five seams, transverse displacements in the form of a wave occur even in the presence of elastic deformations. To illustrate transverse displacement waves, Fig. 4b-d shows the transverse velocities v^1 of the axis of the strip with seams shown in Fig. 4a in elastic deformation, at the beginning of plastic deformation, and in the limiting state. The corresponding longitudinal velocities v^2 of the strip axis are given in Fig. 4e-g.

In the stretching of the strip with seams shown in Fig. 4a the strain in the limiting state of the strip is localized in one seam (Fig. 4d, g). This seam is seen from Fig. 4b, e to differ from the others even in the elastic deformation stage in that the velocity jumps (segments AB and CD in Fig. 4b, e) in this seam are greater than those in other seams. The discrepancy increases with the onset of plastic deformation (Fig. 4c, f). Hence, in the example under consideration one can predict the localization of strains in the limiting state of the strip by the velocity jumps in elastic and initial elastoplastic deformation.

The conclusion that strain localization in the stretching of a strip with seams can be predicted from the velocity jumps in the stages of elastic and initial elastoplastic deformation is confirmed by the results of all the numerical experiments performed by the authors.

REFERENCES

1. V. E. Panin, V. A. Likhachev, and Yu. V. Grinyaev, Structural Levels of Deformation of Solids [in Russian], Nauka, Novosibirsk (1985).
2. V. E. Panin, Yu. V. Grinyaev, V. I. Danilov, et al., Structural Levels of Plastic Deformation and Fracture [in Russian], Nauka, Novosibirsk (1990).
3. V. E. Panin, V. A. Klimenov, S. G. Psakhie, et al., New Materials and Technologies. New Materials and Strengthening Technologies Design [in Russian], VO Nauka, Novosibirsk (1993).
4. S. I. Negreskul, S. G. Psakhie, S. Yu. Korostelev, and V. E. Panin, "Modeling granular media by the method of element dynamics," Preprint No. 39 [in Russian], Akad. Nauk SSSR, Sib. Otd. Tomsk. Nauchn. Tsentr. Tomsk (1989).
5. P. V. Makarov, "Microdynamical theory of plasticity for a medium with internal structure," in: New Methods in Physics and Mechanics of Solids: Proc. Int. Conf. [in Russian], Izd. Tomsk. Univ., Tomsk (1990).
6. O. A. Atamanov, P. V. Makarov, and A. P. Nikolayev, "Mathematical modeling of the process of elastoplastic deformation of polycrystalline aggregates," in: New Methods in Physics and Mechanics of Solids: Proc. Int. Conf. [in Russian], Izd. Tomsk. Univ., Tomsk (1990).
7. G. V. Ianov and V. D. Kurguzov, "Couple-free model of elastoplastic deformation and the limiting state of thin seams," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1994).
8. S. K. Godunov and V. S. Ryaben'kii, Difference Schemes [in Russian], Nauka, Moscow (1977).
9. L. M. Kachanov, Foundations of the Theory of Plasticity [in Russian], Nauka, Moscow (1969).